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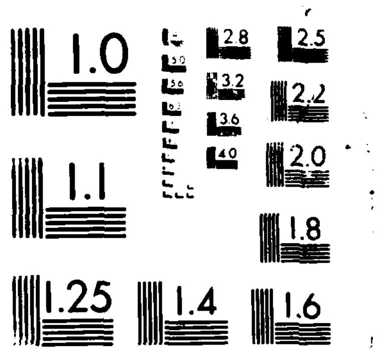

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This project concentrated on issues of nonlinear control design, with an emphasis on digital systems and symbolic methods.

One area of effort was that of studying the effect of the use of sampling on the controllability and observability of nonlinear continuous systems as well as on recently developed linearization techniques. This is closely related to work on discrete-time controllability, also in progress under the grant. Another area dealt with a new method for automatic gain scheduling, for which a computerized design is now available. Experimental results are also described.

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REGULATION OF NONLINEAR AND GENERALIZED LINEAR SYSTEMS

Eduardo D. Sontag, AFOSR-85-0247

Interim Scientific Report, July 1987

Research during this period of the grant has focused on the following subareas of control and system theory: methods for linearization along reference trajectories, the study of the effect of sampling in control problems, nonlinear discrete-time systems, and generalized linear systems. We now discuss the accomplishments and present work in each of these.

§1. Equilinearization.

Already during the first year of the present grant, we proved that a nonlinear controllable plant, under mild technical conditions, admits a precompensator with the following property: along control trajectories joining pairs of states, the composite system (precompensator plus plant) is, up to first order, a parallel connection of integrators. This relates to results of Rugh and of Champetier et. al. on pseudolinearization, but applies to linearizability along trajectories rather that around equilibrium points. A publication submitted last year to the AC Transactions was slightly revised and is now to appear as a full paper there ([7]). A closely related result in continuous-time system controllability insures that the conditions needed in that paper are in fact satisfied under very mild assumptions. This is explained in the paper [6], which was submitted and accepted for publication.

The above result is of interest in the context of the hierarchical approach to control typical in aerospace applications. There, it is often the case that open-loop trajectories are precomputed on the basis of optimal control or other considerations. Regulators are then tuned ("gain-scheduled") to the various operating conditions, or chosen for acceptable performance along the precalculated trajectories. On the basis of the above results, we have been able to provide a design method that allows for a 'universal' construction of regulators along families of desired trajectories. More recent research has dealt with concrete implementations using symbolic algebra packages, -explicit details of which are provided in the conference papers [4], [11], and [12], where various aspects are analyzed,- and with the study of theoretical as well as practical performance of the methodology. At present we have a "computer-aided design method" (actually, a collection of MACSYMA routines) that carry out the necessary constructions. The enclosed graphs, borrowed from [4], show the performance of our method in a problem dealing with the control of the angular velocity of a rotating satellite under the action of a single pair of opposing jets. The first graph shows the desired behavior of one of the coordinates (under a constant control), while the next two display the behavior of this coordinate under open-loop control, assuming an erroneous initial state, and closed-loop control for the same initial state. Note how the closed loop system manages to correct a phase as well as magnitude error. The last graph just compares the open and closed loop errors. The PI has lectured at various places, including NASA-AMES, Wisconsin, Florida, Purdue, and Cornell, as well as given various conference lectures, on this topic.

§2. The effect of sampling.

The second major area of work relates to the topic of *sampling*. When a continuous-time system is regulated by a digital computer, control decisions are often restricted to be taken at fixed times $0, \delta, 2\delta, \ldots$; one calls $\delta > 0$ the *sampling time*. Under what is often called zero-th order hold sampled control, the resulting situation can be modeled through the constraint that the inputs applied be constant on intervals of length δ. It is thus of interest to characterize the preservation of basic system properties when the controls are so restricted. For controllability, this problem motivated the results in the classical paper of Kalman, Ho, and Narendra (1963). This studied the case of linear systems and established that controllability when sampling at frequency $1/\delta$ is preserved if $\delta(\lambda - \mu)$ is not of the form $2k\pi$ for any pair of distinct eigenvalues of the A matrix. In research carried out during the first year, we found a general result which in particular implies, for the large class of bilinear systems, an analogous property; one now needs that $\delta(\lambda + \lambda' - \mu - \mu')$ not be equal to $2k\pi$, k non zero, for any four eigenvalues of the autonomous dynamics matrix. Thus in the bilinear case, one must sample at 4 times (rather than twice) the natural frequencies of the system. The bilinear result is obtained by inducing a linear system on the adjoint representation of a certain Lie algebra associated to the given system. The result was proved in terms of the transitivity, often called also the "weak controllability," property. In recent work we have been able to extend this to the much more interesting *accessibility* property, with the same eigenvalue condition being sufficient. Details are provided in [10], and the treatment is based on the results about discrete-time systems described in the next section.

Closely related to the above topic is another area of recent research by the PI, that of determining the effect of digital implementation on nonlinear control laws for continuous-time smooth systems. In particular, questions have been raised by J.Grizzle and others concerning the effect of such sampling on the currently very popular feedback linearization techniques. The paper [13] deals with this topic. We show there that the possibility of linearization via sampled control introduces certain strong constraints on the form of the controllability Lie algebra of the original system. In particular, we establish in that reference that a system which is feedback linearizable via digital control must be a *graded* system in the sense of our work last year which dealt with the modeling of mechanical systems such as those arising in robotics. This result is obtained through the analysis of a set of necessary and sufficient conditions for sampled feedback linearization, conditions based on Lie-algebraic expansions of certain parametrized vector fields. We have benefited in this work from our interaction with B. Jakubczyk, a visitor at Rutgers during the period September 1986 to December 1987. (Jakubczyk was one of the coauthors of the paper, in 1980, which provided the first set of necessary and sufficient conditions for feedback linearization.)

§3. Discrete-time controllability.

Discrete-time control systems have been studied much less than their continuous counterparts, and their properties diverge considerably from those of the latter, due mainly to the possibility of singularities; this was to a great extent the main topic of the PI's doctoral dissertation 10 years ago. In the early 80's, a paper by B. Jakubczyk introduced the idea of studying *invertible* discrete nonlinear systems, and developed a realization theory which parallels much of the continuous time situation; further work along these lines was carried out in by Fliess, Normand-Cyrot, the PI, and others. Invertible systems are those for which transition maps, (one for each fixed control,) are all (local) diffeomorphisms. Invertibility is of course a priori an extremely strong assumption in the context of general discrete time systems. However, for systems that result from the sampling, as above, of continuous time systems, this assumption is always satisfied. For invertible discrete-time systems, it is possible to give a close analogue of the basic continuous-time orbit theorem of Stephan and Sussmann. Since times are discrete, it is of course not possible to take time derivatives with respect to time, as done classically. Instead, one substitutes derivations with respect to the values of the controls in each interval, the underlying assumption being that there is some sort of manifold structure on the control value set.

In a major paper submitted last year and now published ([5]), we introduced a framework that allows us to prove an abstract orbit theorem, for "smooth actions on manifolds". This yielded as consequences both discrete-time and continuous-time controllability theorems. More interestingly, the theorem also implies a number of other results, including characterizations of "zero-time" orbits of various different types, and an alternative submanifold structure in the continuous-time case (the "input-topology" structure). Certain facts that would appear to be obvious, for instance the second countability of zero-time orbits (but not of arbitrary orbits), turned out to require careful proofs.

During this grant period we have refined considerably these results, and are now able to provide very precise conditions for accessibility analogous to those available in the continuous-time case. This is also joint work with B.Jakubczyk, and a detailed paper ([14]) is in preparation.

§4. Generalized systems.

This work continues the investigation of synthesis problems for parametrized families of systems. There are two main motivations for this line of research. The first is the expectation that parametrized controllers should prove useful in shifting the computational effort to offline preprocessing. This is useful in situations in which the precise values of some system parameters are not known in advance but can be determined on-line, as part of an indirect adaptive control scheme. The second motivation is more purely mathematical: it is natural to ask whether the constructions in control theory can be made "continuous" or "algebraic" in the system parameters.

Consider, for any fixed positive integers n,m, the set of all possible continuous-time systems

$$\dot{x}(t) = Ax(t) + Bu(t),$$

for A an $n \times n$ and B an $n \times m$ real matrix. We know that, if a given pair (A, B) is stabilizable, then there exists a feedback matrix $K = K(A, B)$ such that $A - BK$ is Hurwitz. This construction is continuous, in fact smooth, on the set of all stabilizable pairs (A, B), because a suitable $K(A, B)$ can be found via the solution of a well-posed quadratic optimization problem. What is not known is if a stabilizing $K(A, B)$ can be computed in a more algebraic fashion (the optimization argument depends on the implicit function theorem). We proved in the paper [3] that this can indeed be done provided that *dynamic* feedback be allowed (the precise definition of "algebraic" is via polynomial algebra). This result generalized a previous one by the author regarding stabilization of controllable systems. The proof in the noncontrollable case is completely different, however. In fact, in the controllable case the proof results in static, not dynamic, feedback; whether this can be done in the stabilizable case remains open.

Work in progress is related to pole assignment for parametrized families. We provided in [2] (submitted during the previous grant period) a necessary and sufficient condition for static parametric pole assignability, and started there research into the dynamic case. Current work by the PI and a graduate student being supported during the Summer of 87, attempts to show that in certain cases a number of integrators linear in the plant dimension will suffice. A short paper will probably result from this effort.

§5. Other research directions.

In a recent paper, U.Helmke showed how results of Donaldson in Yang-Mills theory are closely related to system theoretic notions, in particular to what are sometimes called "multirate systems". He then went on to provide a number of results on the topology of the space of framed instantons and of a certain space in which they can be naturally embedded. In [8] we pointed out that it is also possible to view in a natural way these same objects as bilinear systems, or equivalently, via minimal representations of matrix power series. An advantage of this alternative interpretation is that the machinery of Hankel matrices can then be applied to understand the structure of the corresponding moduli space. In particular, we obtained one natural representation of this quotient space as a quasi-affine variety. Explicit equations were given for the moduli space as a quasi-affine variety, using representation theory developed earlier by M.Fliess, the PI, and others.

Finally, we mention our recent work on complexity questions. One of the most important and basic outstanding problems in control theory is that of finding necessary and sufficient conditions for deciding when a continuous-time analytic nonlinear system is (locally or globally) controllable. The goal is to provide some sort of generalization of the classical Kalman controllability rank condition. An early success of this line of research was achieved with the characterization of the *accessibility property*: there is a Lie-algebraic rank condition for deciding if it is possible to reach an open set from a given initial state. When this accessibility rank condition does not hold, all trajectories must remain in a lower-dimensional submanifold of the state space. It is known that local controllability can also be *in principle* checked in terms of linear relations between Lie brackets of the vector fields defining the system, and recent research has succeeded in isolating a number of necessary as well as a number of sufficient explicit conditions for controllability. No complete characterization is yet available, however.

The purpose of [9] was to point out that, whatever necessary and sufficient conditions are eventually found, these are likely to be rather hard to check. One way to quantify this difficulty is in terms of complexity of computation. The relative difficulty of controllability vis a vis the already understood accessibility problem is clarified in the case of the class of systems that can appear as subsystems of bilinear ones. This is a large class of nonlinear systems, including for instance all minimal realizations of finite Volterra series, as well as of course all linear systems. In the context of this class, one can make the precise statement that the accessibility question can be decided in polynomial time, while controllability is NP-hard. Work in progress by the PI attempts to provide tighter complexity estimates.

§6. Remark.

Although not directly part of the grant effort, it is relevant to remark here that the PI has devoted a large amount of effort during this last year to the launching of a new journal, *Mathematics of Control, Signals, and Systems (MCSS)*, to be published by Springer-Verlag. This is intended to be a leading publication in fields that include those in the scope of the present grant. As founder and co-managing editor (with Professor Bradley Dickinson of Princeton's Electrical Engineering Department), the PI expects MCSS to have a major influence in the future development of the area. A related activity was that of starting and co-editing (also with Dickinson) an electronic newsletter, distributed freely through various computer networks, on signal processing and control.

§7. Papers appeared, 15 July 86 to 15 July 87.

[1] "Review of *Multidimensional Systems Theory*," *Linear Alg. and Applications*, **87**(1987): 273-278.

[2] "Comments on 'Some results on pole-placement and reachability'," *Systems and Control Letters* **8**(1986): 79-85.

[3] "Continuous stabilizers and high-gain feedback," *IMA Journal of Mathematical Control and Information*, **3**(1986): 237-253.

[4] "Equilinearization: A simplified derivation and experimental results," *Proc. Conf. Info. Sciences and Systems*, Johns Hopkins University Press, 1987.

[5] "Orbit theorems and sampling," in *Algebraic And Geometric Methods in Nonlinear Control Theory*, M.Fliess and M.Hazewinkel (Eds.), Reidel, Dordrecht, 1986, pp. 441-486.

§8. Papers submitted or in print, 15 July 86 to 15 July 87.

[6] "Finite dimensional open-loop control generators for nonlinear systems," to appear in *Int. J. Control.*

[7] "Controllability and linearized regulation," *IEEE Trans. Automatic Control* **32**(1987): to appear.

[8] "A remark on bilinear systems and moduli spaces of instantons", *Systems and Control Letters*, to appear.

[9] "Controllability is harder to decide than accessibility," submitted.

§9. Papers in preparation, 15 July 86 to 15 July 87.

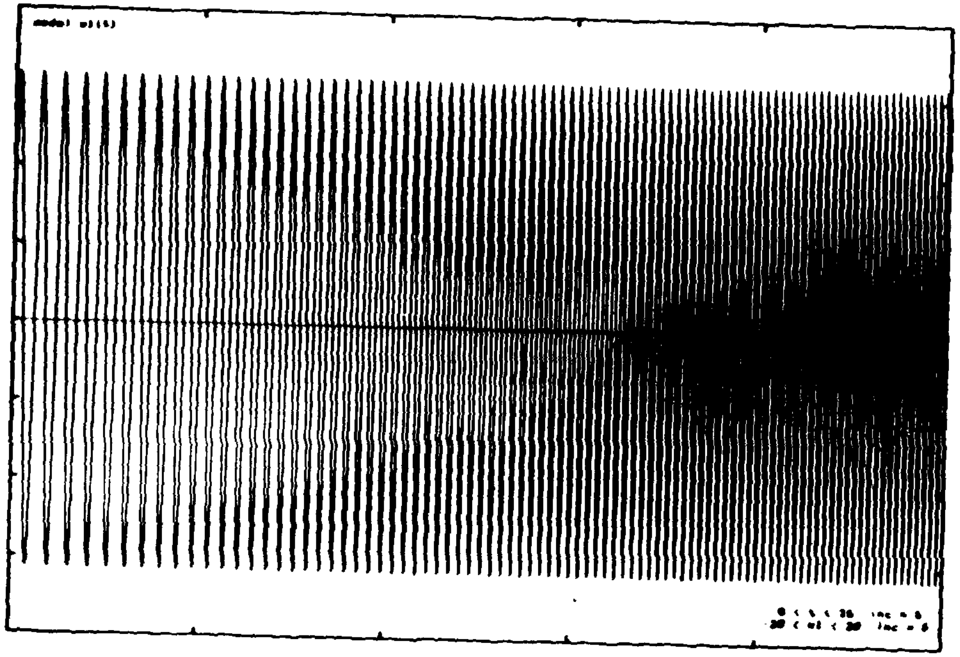
[10] "A Chow property for sampled bilinear systems," to appear in the Proceedings of the *Conference on Mathematical Theory of Networks and Systems*, Tempe, June 1987.

[11] "An explicit construction of the equilinearization controller", to appear in the Proceedings of the *Conference on Mathematical Theory of Networks and Systems*, Tempe, June 1987.

[12] "An approach to the automatic design of first-order controllers along reference trajectories," to appear *Proc. IEEE Conf. Decision and Control, Los Angeles, Dec.1987.*

[13] (With B. Jakubczyk) "The effect of sampling on feedback linearization," to appear in *Proc. IEEE Conf. Decision and Control, Los Angeles, Dec.1987.*

[14] (With B. Jakubczyk) "Controllability of nonlinear discrete-time systems: A Lie-algebraic approach," to be submitted.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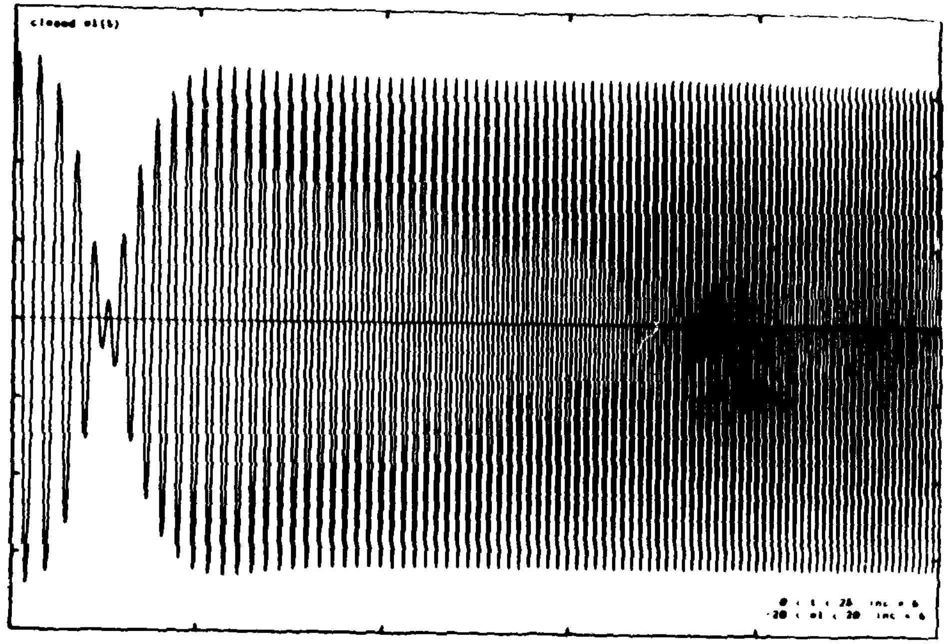
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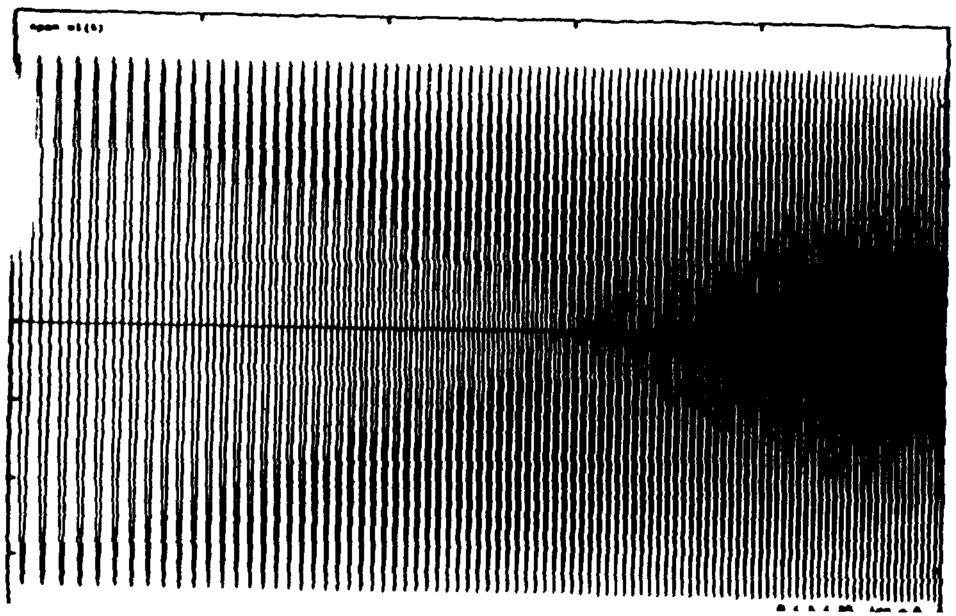
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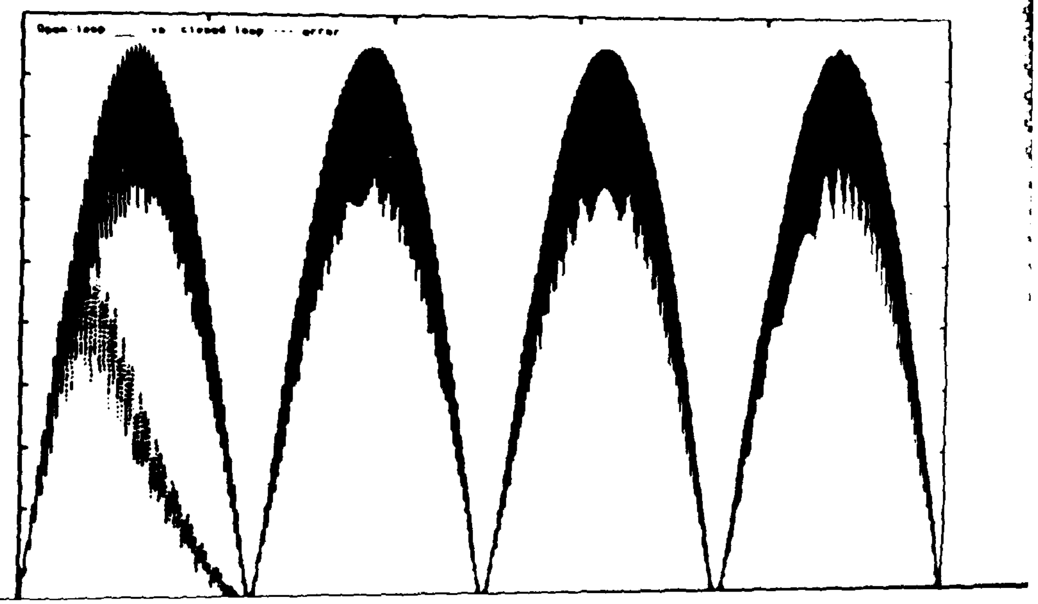
Open-loop __ vs closed loop --- error

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